Spherical astronomy of the Indian Classical Astronomy

（インド古典天文学の球面天文学）

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I. Introduction

I have presented two papers on the history of Indian astronomy in a conference and a seminar at NAOJ, namely, Ôhashi (2011a) on the *Vedāṅga* astronomy, and Ôhashi (2011b) on the planetary theory of the Classical *Siddhānta* astronomy. The present paper is a kind of continuation of these papers. I would like to discuss the spherical astronomy of the Classical *Siddhānta* astronomy, where orthographic projection and plane trigonometry are used.

The history of Indian astronomy can roughly be summarised as follows. [For an overview of Indian astronomy, see Ôhashi (1998) in Japanese or more detailed Ôhashi (2009) in English.]

(i) Indus civilisation period (ca.2600BCE – ca.1900 BCE).

(ii) Vedic period (ca.1500 BCE – ca.500 BCE).

(iii) *Vedāṅga* astronomy period (From sometime between the 6th and 4th centuries BCE up to sometime between the 3rd and 5th centuries CE?).

(iv) Period of the introduction of Greek astrology and astronomy (Sometime around the 3rd and 4th century CE?).

(v) Classical *Siddhānta* period (Classical Hindu astronomy period). (From the end of the 5th century up to the 12th century).

(vi) Coexistent period of the Hindu astronomy and Islamic astronomy (From the 13/14th century up to the 18/19th century).

(vii) Modern period (Coexistent period of the modern astronomy and traditional astronomy). (From the 18/19th century onwards).

Around the 3rd (?) century CE, Greek horoscopic astrology was introduced into India, and around the 4th (?) century CE, Greek mathematical astronomy seems to have been introduced into India. In the Classical Hindu Astronomy period (Classical *Siddhānta* period) (from the end of the 5th century to the 12th century), Indian astronomy did not receive apparent foreign influence, and developed individually.

The Classical Hindu Astronomy period produced several famous astronomers, such as, Āryabhaṭa (b.476), Varāhamihira (6th century), Bhāskara I (fl.629), Brahmagupta (b.598), Lalla (ca.8th or 9th century), Vaṭeśvara (b.880), Mañjula (fl.932), Śrīpati (fl.1039/1056), Bhāskara II (b.1114) etc. And also the anonymous *Sūrya-siddhānta* (ca.10th or 11th century) is a very popular Sanskrit astronomical text of this period. Some of these works are still considered to be authoritative by modern traditional Hindu calendar makers etc. The period during which these classical astronomical works were composed can be called Classical *Siddhānta* period or Classical Hindu Astronomy period. The "*Siddhānta*" is the fundamental treatise of mathematical astronomy in Sanskrit.

II. Three problems ----- Indian spherical astronomy

(II.1) Introduction

The *Siddhānta*s usually consist of two parts, namely, the section of the calculation of planetary position, and the section of spherics. The section of the calculation of planetary position is subdivided into the chapter on the mean motion, the chapter on the true motion, the chapter on the "three problems" (direction, place and time), the sections of lunar and solar eclipses, the sections of the conjunction of planets and stars, the section of heliacal rising and setting, the section of lunar phase etc.

What I am going to present here is the spherical astronomy in the chapter on the "three problems" (chap.III) in the *Sūrya-siddhānta* (ca.10th or 11th century), one of the most popular Sanskrit work on astronomy. There are some versions of the *Sūrya-siddhānta*, and I followed the most popular Raṅganātha's version. Raṅganātha (son of Ballāla) flourished around 1603 AD at Kāśī (=Varanasi =Banaras) in India. He was born in a family of astronomers. He wrote a very popular Sanskrit commentary (1603 AD) on the *Sūrya-siddhānta*. Raṅganātha's son Munīśvara (b.1603 AD) was also an astronomer, and composed a Sanskrit astronomical treatise *Siddhānta-sārva-bhauma* (1646 AD) etc. There are some other astronomers in their relatives also.

Some texts of the *Sūrya-siddhānta* have 51 verses in its chap.III, while some texts have 50 verses in its chap.III, where verse no.32 occurs twice. For example:

The Súrya-siddhánta, an ancient system of Hindu atronomy; with Ranganátha's exposition, the *Gúḍhártha-prakáśaka*, edited by FitsEdward Hall with the assistance of Bápú Deva Śástrin, (Bibliotheca Indica), Calcutta, Asiatic Socirty of Bengal, 1859, has 50 verses in its chap.III.

The Súrya-siddhánta, an ancient system of Hindu atronomy; with Ranganátha's exposition, the *Gúḍhártha-prakáśaka*, Calcutta, Sangbada Jnanaratnakara Press, 1871, has 51 verses in its chap.III.

There are two English translations of the *Sūrya-siddhānta* as follows. The both have detailed English commentary.

Bápú Deva Sástri and Lancelot Wilkinson (tr.): *The Súrya Siddhánta, or an Ancient System of Hindu Astronomy, followed by the Siddhánta Śiromani, translated into English, with extensive explanatory notes*, Bibliotheca Indica Vol.32, Calcutta, Asiatic Society of Bengal, 1861; Reprinted: Amsterdam, Philo Press, 1974. This translation has 50 verses in its chap.III.

Ebenezer Burgess (with the help of the Committee of Publication, particularly W.D. Whitney) (tr.): *The Sûrya Siddhânta, a text-book of Hindu astronomy*, (originally published in the *Journal of the American Oriental Society*, **6**(2), 1860, 141 ~ 498), Reprint edited by Phanindralal Gangooly with an introduction by Prabodhchandra Sengupta, Calcutta, University of Calcutta, 1935; Reprinted: Delhi, Motilal Banarsidass, 1989. This translation has 51 verses in its chap.III.

(II.2) Construction of the gnomon

The *Sūrya-siddhānta* (III.1-4) reads as follows.

"1. On a water-levelled even stone-surface or hard cement, draw a circle with a radius of a desired number of digits (*aṅgula*s) of the gnomon (*śaṅku*)."

"2. At its centre, a gnomon with a height of fixed 12 digits should be erected. The tip of its shadow will touch the circle in the forenoon and afternoon. " (See Fig.1 (a).)

"3. On the circle, mark the two points which are called former and latter. Between them, draw the south-north line by a "fish figure"." (See Fig.1 (b).) (The "two points" are the points Fp and Ap in Fig.1 (a) and (b).)

"4. Between the south and north directions, draw the east-west line by a "fish figure". By fish figures" between the cardinal points, intermediate directions may be determined likewise." (See Fig.1 (c).)

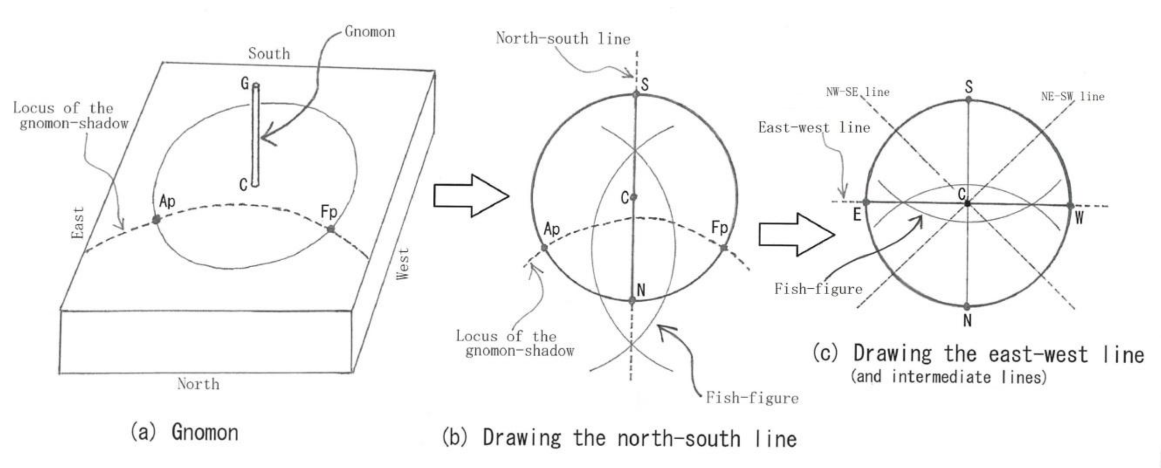

Fig.1, Construction of the gnomon

(II.3) An observed shadow and the celestial sphere

The *Sūrya-siddhānta* (III.5-7) reads as follows.

"5. Draw a circumscribed rectangular by the lines going out from the centre. The desired shadow (*prabhā*) is known to be on the line of the "base" (*bhuja*)." (Here, the "desired shadow" is the segment CB in Fig.2 (a), and the "line of the base" is the line BeBw in Fig.2 (a).)

"6. The (vertical) line (on the celestial sphere) passing through the east and west points is called prime vertical (*sama-maṇḍala*). And also, the six o'clock line (*un-maṇḍala*) and the meridian (*viṣuvan-maṇḍala*) are known." (See Fig.2 (b).)

"7. Establish an east-west line which passes through the tip of the equinoctial midday shadow. The distance between the (tip of) desired shadow and the equinoctial line (established as above) is called "amplitude" (*agrā*)." (The "equinoctial midday shadow" is the segment Cm in Fig.2, and the "east-west line which passes through the tip of the equinoctial midday shadow" is the line MeMw in Fig.2 (a). The "amplitude" is the distance between the lines MeMw and BeBw in Fig.2 (a). Its corresponding segment Gh (in Fig.2 (b)) on the celestial sphere is also called "amplitude".)

The observed shadow changes at every moment, and it corresponds to the hypotenuse of the shadow. The hypotenuse is obtained by the Pythagorean theorem:

$(\text{Shadow-hypotenuse})^2 = (\text{Gnomon-height})^2 + (\text{Shadow-length})^2$.

The gnomon-height is usually considered to be 12 digits.

The Radius (= *R*) of the celestial sphere is usually considered to be 3438, because 360×60 minutes divided by 2π are about 3438 minutes.

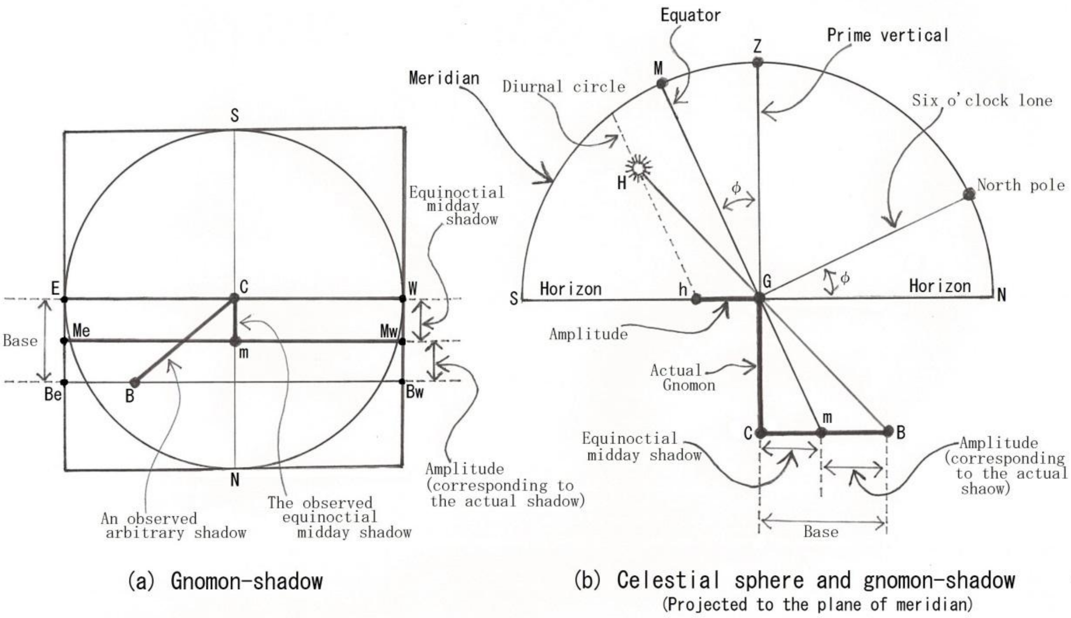

Fig.2, Observed shadow and celestial sphere

(II.4) Midday shadows

The *Sūrya-siddhānta* (III.12cd-13ab) tells to obtain the "equinoctial midday shadow" (*viṣuvat-prabhā*) (the segment Cm in Fig.3 (b)) at the observer's place.

The *Sūrya-siddhānta* (III.13cd-14ab) tells to convert the "equinoctial midday shadow" (the segment Cm) and the "gnomon height" (the segment CG in Fig.3 (b)) to the celestial sphere, using shadow hypotenuse (the segment Gm in Fig.3 (b)), and obtain the co-latitude (90°－φ) and latitude (φ) of the observer as follow.

$$\frac{R}{Shadow-hypotenuse\ (Gm)} \times \text{Gnomon-height (CG)} = R\text{sine of the co-latitude.}$$

$$\frac{R}{Shadow-hypotenuce\ (Gm)} \times \text{Shadow-length (Cm)} = R\text{sine of the latitude.}$$

They are converted into the arcs of co-latitude and latitude.

The *Sūrya-siddhānta* (III.14cd-16) tells the relationship between the desired midday shadow, called "base" (*bhuja*) (the segment Cd in Fig.3 (b)), the Sun's midday zenith distance (ζ) and its declination (δ). The "shadow-hypotenuse" is the segment Gd in Fig.3 (b).

$$\frac{R}{Shadow-hypotenuse\ (Gd)} \times \text{"base" (Cd)} = R\text{sine of the Sun's zenith distance } (= R\sin\zeta).$$

It is converted into the arc of the Sun's midday zenith distance (ζ).

If the Sun's midday zenith distance and declination are in the opposite directions, φ =ζ + δ.

If the Sun's midday zenith distance and declination are in the same direction, φ =|ζ－δ|.

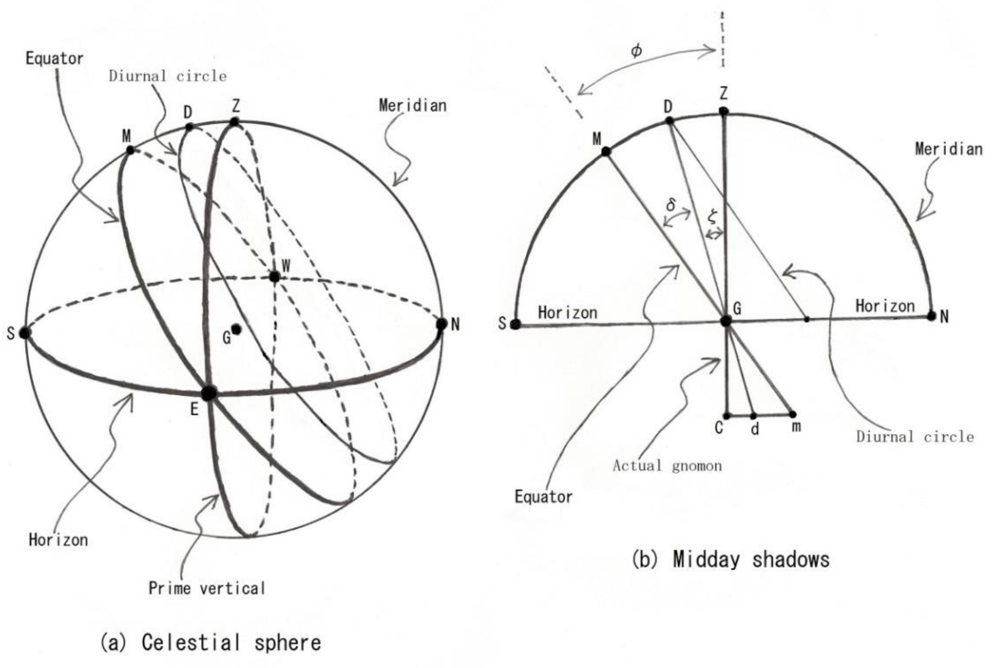

Fig.3, Midday shadows

(II.5) Solar longitude

The *Sūrya-siddhānta* (III.17cd-19) tells to obtain the Sun's longitude (λ) from its declination (δ), when the obliquity of the ecliptic (ε) is known. In the case of the first quadrant, it is as follows.

$$\frac{R \times Rsin\delta}{Rsin\varepsilon} = R\sin\lambda.$$

This formula can be understood as follows. The segment gKc in Fig.4 (a), which is projected as the segment GKc in Fig.4 (b), is equal to *R*sinλ. And also, the segment KqKc is equal to *R*sinδ. As the triangles gKcKq and GLcLq are similar, and the segment LqLc is equal to *R*sinε, the above equation is obtained.

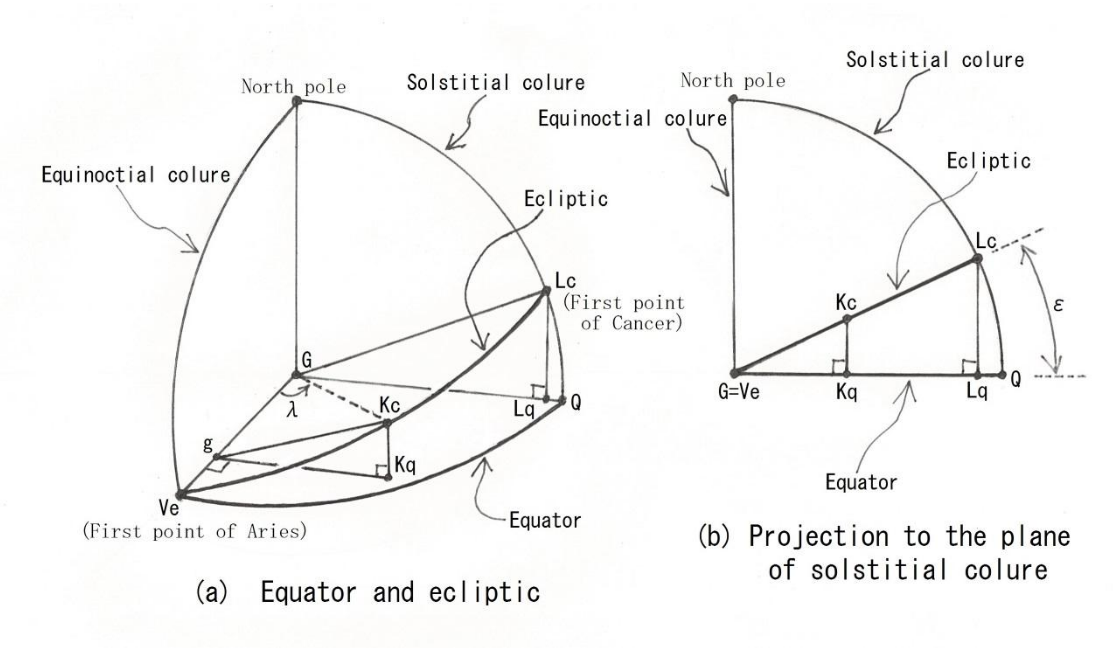

Fig.4, Solar longitude

(II.6) Time and shadow

The "ascensional difference" (*cara*) corresponds to the arc between the point of sunrise and the six o'clock line. The "*R*sine of the ascensional difference" (*cara-jyā*) (the segment GAe in Fig.5 (c)) is obtained as follows. (The angle AGB is equal to the observer's latitude (φ).)

GB = *R*sinδ,

AB = GB × $\frac{Rsin\varphi}{Rcos\varphi}$ (= *R*sinδ×tanφ).

"Cara-jyā" (GAe) = $\frac{R}{r}$ AB,

where *r* is the rdius (BD) of the diurnal circle, that is *r* = *R*cosδ.

The *Sūrya-siddhānta* (III.34cd-36 in the texts of 51 verses) tells that the sum or difference of the

Radius and the “*cara-jyā*“ is the “day-measure“ (*antyā*) (the segment AeM in Fig.5 (c)):

When the Sun’s declination is north: “Day-measure” (*antyā*) (AeM) = R + “cara-jyā” (GAe).

When the sun’s declination is south: “Day-measure” (*antyā*) (AeM) = R − “cara-jyā” (GAe).

The “day-measure“ (*antyā*) (AeM) diminished by the “*R*versed-sine (*utkrama-jyā*) of the hour angle (*nata*) of the Sun” (the segment MHe in Fig.5 (c)), then multiplied by the “day-radius” (*ahorātra-ardha*) (r = BD), and divided by the Radius (R = GM) is the “divisor” (*cheda*) (the segment AHn in Fig.5 (c)):

“Divisor” (*cheda*) (AHn) = $\frac{r}{R}$ (“day-measure” − “*R*versed-sine of the hour angle”).

Then, the “divisor” (the segment AHn in Fig.5 (c) or AHm in Fig.5 (b)) multiplied by the “*R*sine of co-latitude” (*lamba-jyā*) (the segment MMg), and divided by the Radius (the segment GM) is the “*R*sine of the Sun’s altitude” (*śaṅku*) (the segment HnTn in Fig.5 (c) or HmTm in Fig.5 (b)):

$$R\text{sine of the Sun's altitude (HnTn or HmTm)} = \frac{\text{"divisor"} \times \text{"Rsine of co-latitude"}}{R}$$

By the Pythagorean theorem:

$(\text{Radius})^2 - (R\text{sine of the Sun's altitude})^2 = (\text{“}R\text{sine of the Sun's zenith distance” } (d\d{r}g\text{-}jy\bar{a}))^2$.

The actual gnomon-shadow can be calculated from this value.

The *Sūrya-siddhānta* (III.37-39 in the texts of 51 verses) explains the method to obtain time from the observed shadow. The method is just opposite to the above procedure.

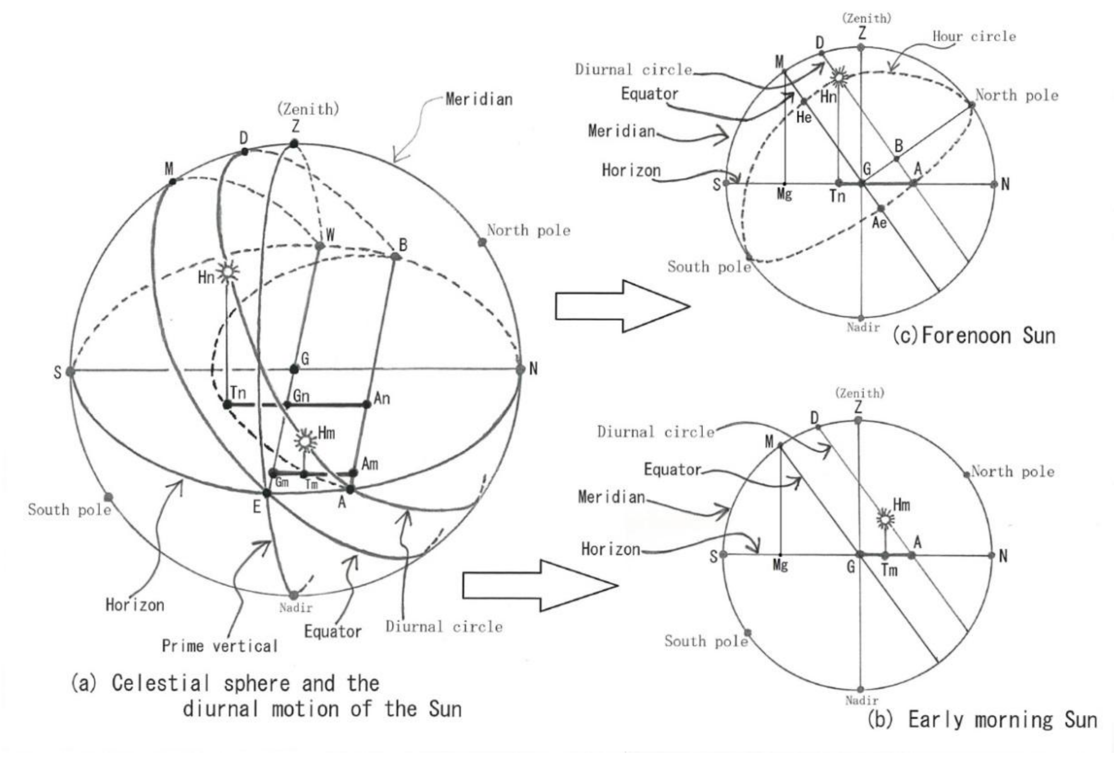

Fig.5, Time and shadw

(II.7) Zodiacal signs

The *Sūrya-siddhānta* (III.42 in the texts of 51 verses) explains the method to calculate the right ascension ("*laṅkā-udaya*", which means the ascension at Laṅkā which is supposed to be on the equator) of the first points of the zodiacal signs. The method is:

$$R\sin\alpha = \frac{R sin\lambda \times R cos\varepsilon}{R cos\delta}.$$

From its result, the arcs of the right ascensions of the first points of zodiacal signs can be calculated.

This equation can be understood as follows. In the Fig.6 (a), the segment Q1q1 is the *R*sine of the right ascension (α) of the first point of Taurus. The segment GC1 is *R*cosine of the declination (δ) of the first point of Taurus. Now, the segment GC3 is the *R*cosine of the obliquity of ecliptic (ε). The segment C1g1 is the *R*sine of the longitude (λ) of the first point of Taurus (λ = 30°) projected to the plane of the equator, that is:

The segment C1g1 = $R\sin\lambda \times \frac{R cos\varepsilon}{R}$.

Considering the above equations,

*R*sinα (the segment Q1q1) = C1g1 × Radius ÷ GC1, that is:

$$R\sin\alpha = R\sin\lambda \times \frac{R cos\varepsilon}{R} \times \frac{R}{R cos\delta} = \frac{R sin\lambda \times R cos\varepsilon}{R cos\delta}.$$

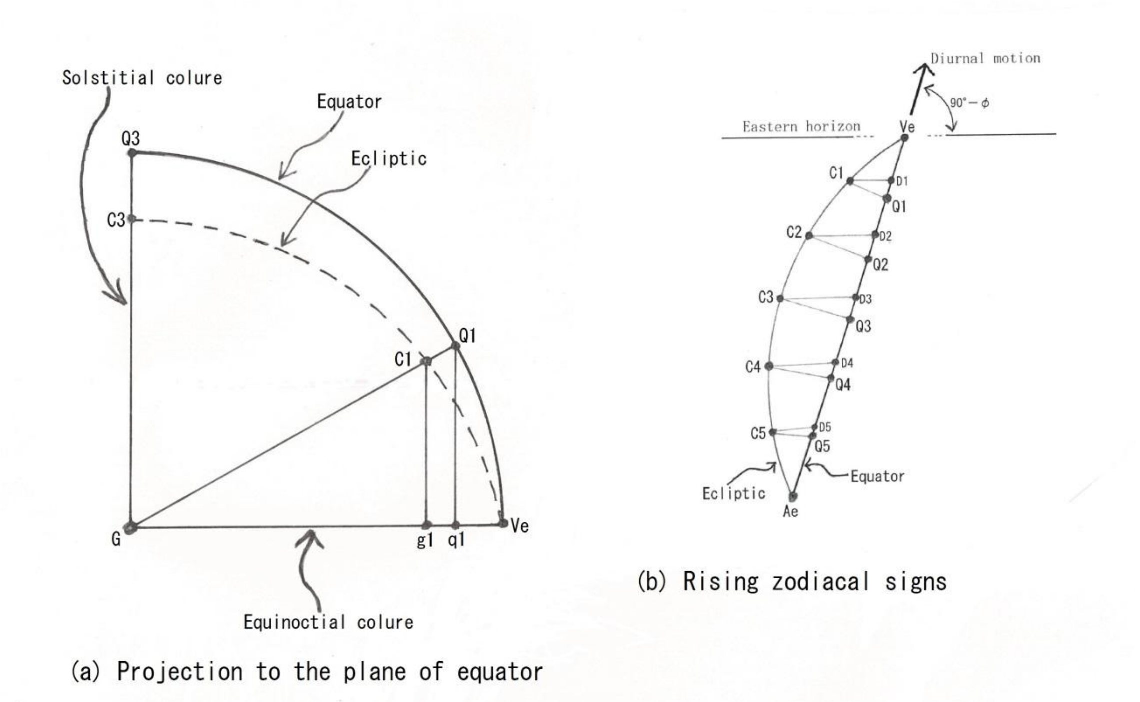

Fig.6, Zodiacal signs

The *Sūrya-siddhānta* also tells related topics conceding the oblique ascension of the observer's latitude, the rising point of the ecliptic called "lagna", etc. In Fig.6 (b), the point Ve is the first point of Aries, the point C1 is the first point of Taurus, etc., and the segment VeQ1 correspondss to the right ascension of the first point of Taurus, etc., and the segment VeD1 corresponds to the oblique ascension of the first point of Taurus at the latitude of observer, etc.

III. Conclusion

We have seen that several problems of spherical astronomy can be solved by plane trigonometry. They can be understood easily, and sometime they will be useful in astronomy education.

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