A study of Parameśvara's record of eclipses

Aditya Kolachana and K. Ramasubramanian

Cell for Indian Science and Technology in Sanskrit

Dept. of Humanities and Social Sciences

Indian Institute of Technology Bombay, India

1. Introduction

India has a hallowed history of mathematics and astronomy. Starting with Āryabhaṭa (5th CE), there were several illustrious names like Varāhamihira, Brahmagupta, Bhāskara – I, Lallācārya, Mahāvīrācārya, Śrīdhara, Śrīpati, Bhāskarācārya (12th CE), and so on, who made significant contributions in the areas of arithmetic, geometry, algebra, mensuration and the like.

During the 14th to the 16th centuries, generally referred to as the golden age of mathematics in India, the study of astronomy and mathematics really flourished in the Malabar region of Kerala, on the banks of the Nilā river. A succession of scholars, collectively referred to as the Kerala School of Mathematics, made remarkable contributions to the advancement of the field that is referred to as infinitesimal calculus today. This school was founded by a gifted mathematician and astronomer named Mādhava who is best known for inventing the infinite series for pi, and other trigonometric functions for the first time in the history of mathematics. Some of the noted astronomers and mathematicians who followed Mādhava include Parameśvara, Dāmodara, Nīlakaṇṭha, Jyeṣṭhadeva, Śaṅkara Vāriyar and so on.

The primary aim of this paper is to make a systematic study of the eclipses recorded by Parameśvara, who was a direct disciple of the illustrious Mādhava. Since the details of the occurrence of eclipses are presented by Parameśvara using the *Kali-ahargaṇa* calendar (day count from the beginning of Kali epoch starting 17th / 18th February, 3102 BCE), we had to develop necessary tools for the conversion of dates between Indian and Gregorian calendars in order to study these historical eclipses. This study, as may be seen from sections 5 and 6 of this paper, in addition to contributing to the pages of the history of science, could also facilitate further research in arriving at better estimates of fluctuations in the length of day (ΔT) – which is a very interesting and challenging problem pursued by astronomers in different nations across the world.

2. Parameśvara and his contributions

Besides being a disciple of the great savant Mādhava, Parameśvara (1360 – 1455) himself was a prolific writer and is credited with several works in astronomy and astrology. He hailed from the village Aśvatthagrāma situated at the confluence of the Nilā river and the Arabian sea[1]. Understandably, Parameśvara time and again refers to the banks of the Nilā river (निलातटे) while presenting the details of the eclipses observed by him (see section 5).

The works of Parameśvara include original writings (such as *Dṛgganita* and *Goladīpikā*), as well as detailed commentaries on other major works. Some of his well known commentaries include *Bhaṭadīpikā* – a commentary on the *Āryabhaṭīya* of Āryabhaṭa, and *Siddhāntadīpikā* – a super-commentary on Govindasvāmin's *Mahābhāskarīya-bhāṣya*. He has also authored commentaries on *Laghumānasa*, *Sūryasiddhānta*, *Līlāvatī* and the like, some of which are yet to be published.

In addition to being an expert theoretician, Parameśvara was also an avid practical astronomer. Being situated on the vast sandy expanse of the banks of the Nilā river as it converges with the Arabian sea, he seems to have carried out astronomical observations for over 55 years, and also meticulously noted down the details of the occurrence of a variety of astronomical phenomena. In order to elucidate the principles behind the computation of eclipses, Parameśvara composed three short treatises exclusively on eclipses: *Grahaṇāṣṭakam*, *Grahaṇamaṇḍana,* and *Grahaṇa-nyāya-dīpikā*. In his *Siddhāntadīpikā*, he records 13 eclipses (8 solar and 5 lunar) which he personally observed, along with a brief description of their characteristics. Nīlakaṇṭha, another famous mathematician and astronomer from the Kerala school, and grand-disciple of Parameśvara, also cites these observations of Parameśvara in his monumental treatise *Āryabhaṭīya-bhāṣya* (a commentary on the *Āryabhaṭīya)*. In the same work, Nīlakaṇṭha extolls Parameśvara as *Golavid* (one who has excelled in spherics). He observes[2]:

> "परमेश्वराचार्येण पुनः ग्रहणग्रहयोगादिकं यन्त्रैः पञ्चपञ्चाशद्वर्षकालं सम्यक् परीक्षितम्। … तस्य गोलवित्तमत्वं च गोलदीपिकादिभिः तत्कृतैः प्रबन्धविशेषैः ज्ञायते। अत अन्येषां करणानां स्वस्वकाले यावत्सूक्ष्मत्वं ततोऽप्येतत्कृतस्य दृग्गणिताख्यस्य सूक्ष्मत्वमुपपद्यते। अपि च

1 The renowned indologist KV Sarma (in his D.Litt Thesis Volume I, p.200.) has identified the modern names of the village and river as Alattur and Bharathappuzha respectively.

2 Pillai 1957, p.154.

परमेश्वरोक्तमन्दोच्चपरिध्यादीनाम् आर्यभटोक्तेभ्यः सूक्ष्मत्वमिदानीमपि ग्रहयोगादिष्वस्माभिर्ज्ञायते।"

> "Eclipses, conjunctions of planets etc. were examined with the help of instruments by the great preceptor Parameśvara for 55 years … His profundity in spherical astronomy is evident from some of the treatises like *Goladīpikā* composed by him. His work *Dṛggaṇita* turns out to be more accurate than other *karaṇa* (handbook) works of his time. Moreover, even today (ca. 16th century), parameters of the circumference of epicycle etc. given by Parameśvara are more accurate than those stated by Āryabhaṭa as evident from the calculation of the conjunction of planets etc."

In yet another work *Siddhānta-darpaṇa-vyākhyā*, in connection with specifying the magnitude of the precision of the equinox (*ayanāṃśa*), Nīlakaṇṭha again adores Parameśvara as his *paramaguru* (g*uru's guru*) with great reverence[3]:

"...यतो भार्गवपरमेश्वराचार्येण अस्मत्परमगुरुणा चलांशास्स्वं (४५३६) इति कल्यब्दे परीक्ष्य पञ्चदशांशपूर्तिनिर्णीता।"

> "...since our *paramaguru* Bhārgava Parameśvara in the Kali year 4536 has specified the magnitude of the precession of equinoxes as 15 degrees after experimentation."

The above citations are provided not only to make it amply clear that Parameśvara was held in very high regard, but also to show that he was considered an authority in the field of astronomy. But for Parameśvara being an astronomer par-excellence, the great savant Nīlakaṇṭha would not have quoted him while discussing the problem of precession of equinoxes, and deciding on the value of *ayanāṃśa*. In fact, the works of Parameśvara have strongly influenced the works of subsequent mathematicians and astronomers – particularly his *Dṛggaṇita*, which came as a revision of the older *Parahita* system of computation.

3. The methodology adopted for our study

In order to study Parameśvara's record of eclipses, as mentioned earlier we had to first convert the recorded Indian dates into Gregorian dates, and then find out whether eclipses actually occurred on those days. To achieve this, we first developed a date conversion tool to convert Indian dates specified in *Kali-ahargaṇa* or *Śaka* systems to dates based on the Gregorian calendar. We also wrote a software application using modern astronomical algorithms to predict the occurrence of historical

3 Sarma 1977, p.17.

solar and lunar eclipses. Finally, we compared the dates given by Parameśvara against the results that were obtained using our software application as well as the database of historical eclipses maintained by the National Aeronautics and Space Administration (NASA), USA. The software applications referred to above were built using the Python programming language. These applications are briefly described below.

Application I: Date conversion tool

The conversion of dates between the Indian luni-solar calendar (generally specified in terms of *Śaka* year) and the Gregorian calendar is not a trivial problem. This is because of the introduction of what are known as *adhimāsa* (intercalary months) and *kṣayamāsa* (omitted months) in the Indian calendrical system. Twelve lunar months constitute only ~354 days (~29.54 * 12), while a solar year is ~365 days. Therefore, a lunar year lags a solar year by approximately 11 days. In order to see that the lunar calendar does not lag behind the solar calendar, an *adhimāsa* is added to the lunar calendar roughly once every 2.75 years on an average. However, this introduction is not done arbitrarily. It is based on the occurrence of a certain physical phenomenon, namely a *saura-saṅkramaṇa* (transit of the sun from one zodiac to another). An *adhimāsa* is said to occur when a *saura-saṅkramaṇa* does not occur within a lunar month. Similarly, a lunar month is omitted in the Indian calendar when two *saura-saṅkramaṇas* occur within a lunar month. Finding the precise occurrence of these two types of months is not straightforward because they do not occur periodically, and also crucially depends upon calculation of the instantaneous rate of motion of the sun and the moon as seen from the Earth. However, we have written appropriate code to compute their occurrence as accurately as possible.

Taking a contemporary date as reference, and finding the correspondence between the Indian and Gregorian calendars for this day, our tool starts back computing, and adjusts for the intercalary as well as omitted months. Accordingly, the tool accurately maps the dates from the Indian calendar to the Gregorian calendar and vice-versa.

Application II: Program to compute eclipses

This application was developed in order to predict the occurrence as well as the duration of solar and lunar eclipses and is based on the algorithm given by Peter Duffet-Smith[4] for this purpose. The classic text on spherical astronomy by W.M. Smart[5] was also consulted.

4 Duffet-Smith 1981.
5 Smart 1949.

Any astronomical computation commences with the specification of a set of parameters that include the initial positions of the planets, as well as other orbital elements for a particular day. This is chosen as the reference day or epoch for all further calculations. In our case, we have chosen the orbital elements of the Sun and Moon on January 1, 1980 as our standard reference, and then proceeded to calculate their mean positions on any desired day using their mean motions. We then apply corrections to the mean positions to arrive at the true longitudes of the sun and the moon. In the case of the sun, we applied only the equation of center correction (known as *mandasaṃskāra* in Indian astronomy) in order to arrive at the true position of the sun. In the case of the moon, besides the equation of center, corrections for evection, annual equation, third and fourth corrections, and variation are applied to arrive at its true longitude.

By testing for the occurrence of conjunction and opposition of the sun and the moon, we predict the occurrence of new moon and full moon days respectively. If at the time of their conjunction or opposition, the distance of the moon from its node is within a prescribed range (known as ecliptic limits), then a solar or lunar eclipse is said to occur. We have developed the necessary code to predict the occurrence of these events, and have also extensively tested its accuracy.

4. Verifying Parameśvara's record of eclipses in *Siddhāntadīpikā*

It was noted earlier that Parameśvara has recorded the details of 13 eclipses that were observed by him during his lifetime in his *Siddhāntadīpikā.* These eclipses span over a period of more than three decades (from 1398 to 1432) as can be seen in Table 1. The description provided by Parameśvara invariably includes the date of occurrence of the eclipse in *Kali-ahargaṇa*, as well as the type of the eclipse (solar or lunar). The other details provided by him vary from eclipse to eclipse. These may include the nature of the eclipse (full, partial), the location of its appearance, the time and duration of occurrence, and so on. Interestingly, all these descriptions are given in the form of verses composed in the *anuṣṭubh* metre. The early part of his record describes eight solar eclipses – not necessarily in chronological sequence – along with a very brief description of their characteristics. The last four verses describe five lunar eclipses.

Having converted the dates of the occurrence of eclipses specified by Parameśvara in *Kali-ahargaṇa* into the Gregorian calendrical dates, we checked for the occurrence of eclipses on those dates using both our eclipse tool as well as the NASA database. The details of the comparison are listed in Table 1. It may be noted that there is a complete concurrence between the dates specified in the *Siddhāntadīpikā* and the ones obtained with our eclipse tool, as well as the NASA database. This clearly indicates that the record of eclipses presented by Parameśvara forms a reliable historical record which is sought after by the astronomers who are working on arriving at a better expression for ΔT.

Table 1: Comparison of Parameśvara's record with our eclipse tool and NASA database

Parameśvara's record			**Eclipse Tool**	**NASA database**
Kali-ahargaṇa	Type of Eclipse	Converted Date (CE)	Date (CE)	Date (CE)
16,43,524	Solar	9th Nov, 1398	9th Nov, 1398	9th Nov, 1398
16,47,156	Solar	19th Oct, 1408	19th Oct, 1408	19th Oct, 1408
16,48,722	Solar	1st Feb, 1413	1st Feb, 1413	1st Feb, 1413
16,52,000	Solar	23rd Jan, 1422	23rd Jan, 1422	23rd Jan, 1422
16,53,387	Solar	10th Nov, 1425	10th Nov, 1425	10th Nov, 1425
16,55,130	Solar	19th Aug, 1430	19th Aug, 1430	19th Aug, 1430
16,55,484	Solar	8th Aug, 1431	8th Aug, 1431	8th Aug, 1431
16,55,662	Solar	2nd Feb, 1432	2nd Feb, 1432	2nd Feb, 1432
16,52,694	Lunar	17th Dec, 1423	17th Dec, 1423	17th Dec, 1423
16,53,403	Lunar	25th Nov, 1425	25th Nov, 1425	25th Nov, 1425
16,54,614	Lunar	20th Mar, 1429	20th Mar, 1429	20th Mar, 1429
16,55,293	Lunar	28th Jan, 1431	28th Jan, 1431	28th Jan, 1431
16,55,647	Lunar	17th Jan, 1432	17th Jan, 1432	17th Jan, 1432

5. A sample of eclipses from the record of Parameśvara

As indicated above, since Parameśvara's record of eclipses is reliable, it can be used for various historic and scientific studies including the calculation of a better expression for ΔT, which requires a reliable historic record. Of the 13 eclipses recorded by Parameśvara, the following three solar eclipses may be of interest for calculating ΔT, since they furnish details such as the location of visibility of the eclipse, the time of occurrence of the eclipse in that locality, and so on. Parameśvara's original verses from the *Siddhāntadīpikā*[6] recording these eclipses are presented below, along with our translation.

6 Kuppanna Sastri 1957, pp. 329 – 331.

1. Solar eclipse of 19th October, 1408

द्युगणे रसतिथ्यद्रिवेदषड्भूमिसम्मिते।

मुक्तेऽर्केऽस्तमयो दृष्टः नाडीपादोऽस्ति वान्तरे।।

On *Kali-ahargaṇa* corresponding to 16,47,156 the sun was observed to be setting when it was released (last contact) from the eclipse, or there was a difference of a quarter of a *nāḍī* (~6 minutes) between them (last contact and sunset).

2. Solar eclipse of 10th November, 1425

द्युगणे सप्तनागाग्निगुणेषुरसभूमिते।

गोकर्णे ग्रहणं भानोः दृष्टं नात्र निलातटे।।

On *Kali-ahargaṇa* corresponding to 16,53,387 a solar eclipse was observed in Gokarṇa, but not here on the banks of the Nilā river.

3. Solar eclipse of 19th August, 1430

शून्याग्निभूशरेष्वङ्गभूतुल्ये द्युगणे रवेः।

गोकर्णे ग्रहणं दृष्टं निलाब्ध्योः संगमेऽत्र न।।

प्रोक्ते दिनेऽपि बिंबस्य पार्श्वे वर्णस्य भेदनम्।

कैश्चित् कुमारैरत्रापि कल्पितं वा निलातटे।।

On *Kali-ahargaṇa* corresponding to 16,55,130 a solar eclipse was observed in Gokarṇa, but not at the confluence of the Nilā river with the ocean. On the said day also, some boys claimed to have observed a change in shade on one side of the solar orb here too, on the banks of Nilā river.

6. The relevance of this record for ΔT calculations

Of late, Morrison and Stephenson in one of their recent publications[7], have explained the importance of historical eclipse records in arriving at a better estimate of ΔT. The value of ΔT, which takes into account the effect of variation in the Earth's rate of rotation, is very crucial in

7 Morrison et. al. 2004.

calculating eclipses in the remote past. According to them, as of now, there seems to be “... a degree of confusion and misapprehension among the historians of astronomy over the choice of values of ΔT...”. In this connection, it was felt that the 'reliable record' of eclipses given by Parameśvara could be of some relevance.

In the first of the three examples presented above, it is stated that the eclipse occurred close to sunset. Since it is also stated that the eclipse was over just 6 minutes before sunset, this could be particularly useful for ΔT calculations. The path traced by this eclipse is shown in Figure 1[8].

In the second example, Parameśvara states that the eclipse was observed in Gokarṇa (14.55° N, 74.3167° E) but not on the banks of the Nilā river (10.77° N, 75.9° E). This gives us latitude and longitude limits for the path of the eclipse which also may prove to be useful in the calculation of ΔT. Figure 2 shows the path of this eclipse.

Similarly, in the third example it is stated that this eclipse too was observed in Gokarṇa but not at the confluence of the Nilā river with the ocean. Therefore, we may use the same longitude and latitude limits as above in trying to arrive at a better estimate of ΔT. Figure 3 shows the path traced by this eclipse.

A more detailed investigation into Parameśvara's record, as well as other records of historical eclipses found in certain inscriptions in India could further assist in arriving at better estimates of ΔT.

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8 Source of all figures: NASA. Weblink given in the list of references.

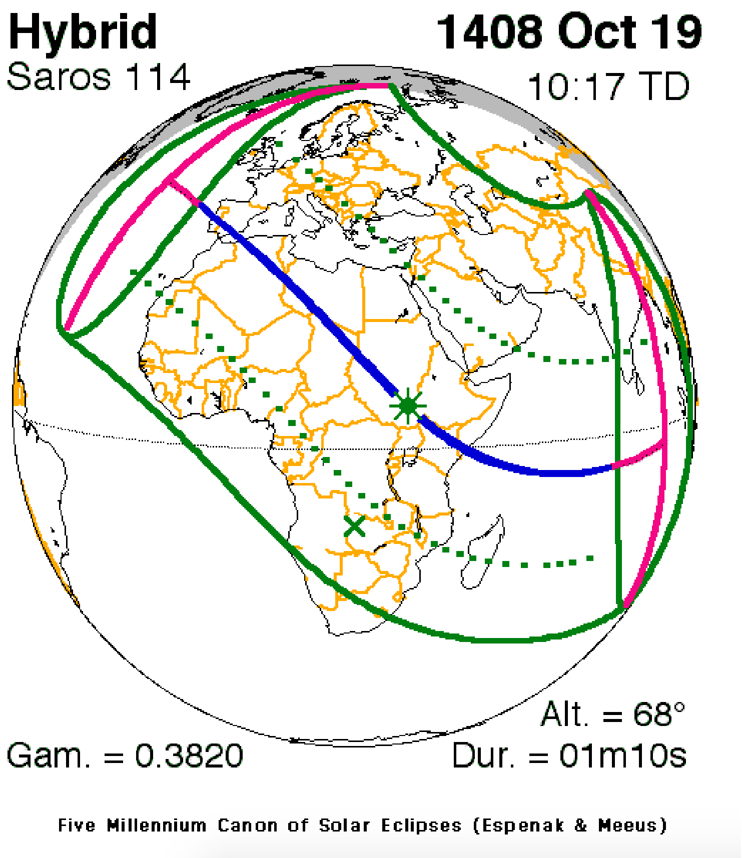

Figure 1: Path of solar eclipse on 19th October, 1408

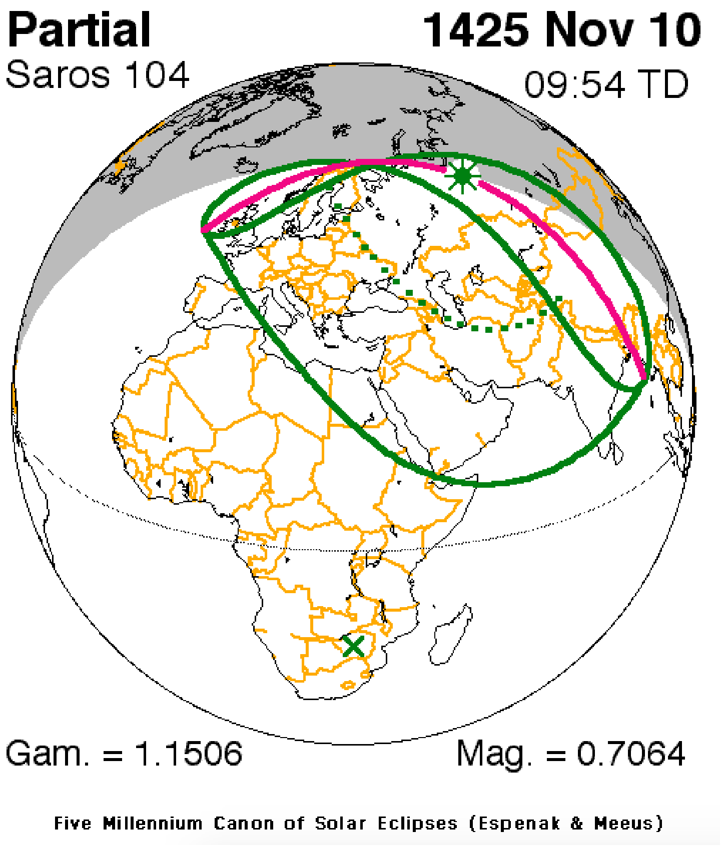

Figure 2: Path of solar eclipse on 10th November 1425

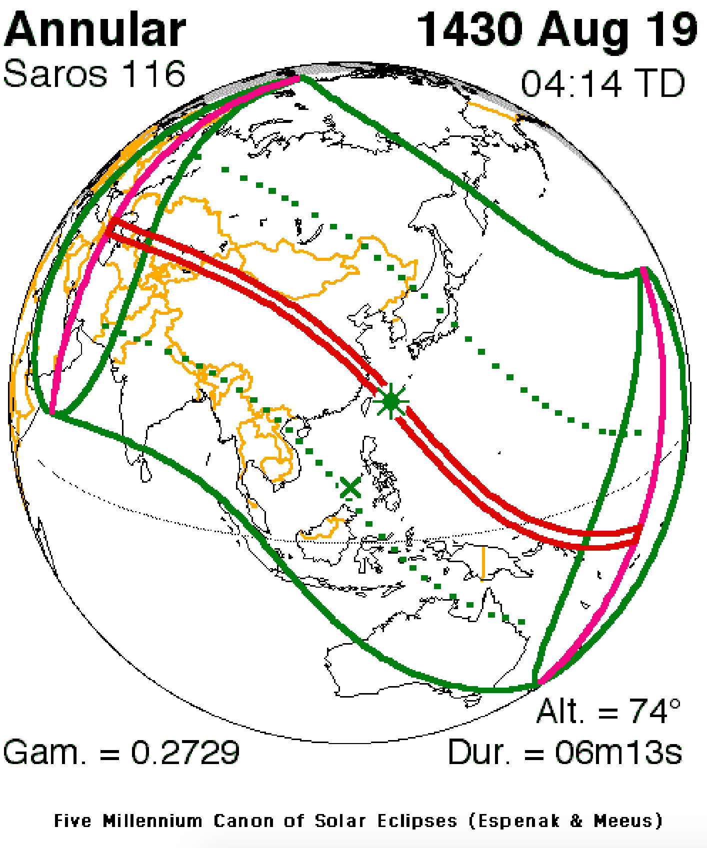

Figure 3: Path of solar eclipse on 19th August, 1430

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