Length	L	300 m
Free spectral range	FSR	500 kHz
Input mirror transmission	t_1^2	0.7%
End mirror transmission	t_2^2	2.9%
Finesse	F	172
Linewidth (FSR/Finesse)	dv	2.9 kHz
Cavity pole		1.45 kHz
modulation depth	m	0.1 rad
Modulation frequency	Ω	$2\pi \cdot 90$ Hz
Input Power	P_0	0.25 mW

TABLE 1. Filter cavity parameters for green light

1. Optical gain computation

In Tab ?? are reported the parameter used in the following computation

The modulated input beam has the form:

$$E_{in} = Ee^{i(\omega t + m \sin \Omega t)}$$

which can be expanded in terms of Bessel functions as

$$E_{in} = E[J_0(m)e^{i\omega t} + J_1(m)e^{i(\omega+\Omega)t)} - J_1(m)e^{i(\omega-\Omega)t}]$$

This shows that the phase modulation has created two sidebands at a distance Ω from the carrier, whose amplitudes depends on the modulation depth.

The reflectivity of the filter cavity for the green light can be written as

$$R_{cav} = \frac{-r_1 + r_2 \exp\left(i\frac{\omega}{FSR}\right)}{1 - r_1 r_2 \exp\left(i\frac{\omega}{FSR}\right)}$$

The beam reflected from the cavity is :

$$E_{ref} = E[R_{\text{cav}}(\omega)J_0(m)e^{i\omega t} + R_{\text{cav}}(\omega+\Omega)J_1(m)e^{i(\omega+\Omega)t} - R_{\text{cav}}(\omega-\Omega)J_1(m)e^{i(\omega-\Omega)t}]$$

From that we can compute the power impinging on the photodiode.

$$\begin{aligned} P_{ref} =& P_c|R_{\text{cav}}(\omega)|^2 + P_s|R_{\text{cav}}(\omega+\Omega)|^2 + |R_{\text{cav}}(\omega-\Omega)|^2 \\ &+ 2\sqrt{P_c P_s}[\text{Re}[R_{\text{phs}}]\cos\Omega t + \text{Im}[R_{\text{phs}}]\sin\Omega t] \end{aligned}$$

with $P_c = P_0 J_0^2(m)$, $P_s = P_0 J_1^2(m)$ and $R_{\text{phs}} = R_{\text{cav}}(\omega)R^*_{\text{cav}}(\omega+\Omega) - R^*_{\text{cav}}(\omega)R_{\text{cav}}(\omega-\Omega)$

The reflected power is composed by a DC part and two terms oscillating at the modulation frequency, whose amplitude is proportional respectively to the imaginary and real part of the function R_{phs} and thus keep the important information on the phase of the reflected beam. These terms arise from the beating of the carriers and the sidebands. Here we neglect the beating of each sidebands whit the other one. In order to extract the phase information contained in the oscillating terms, the signal measured by the photodiode is

PD Photosensitivity	$16 \cdot 10^3$ V/A
PD transipendence	0.25 A/V
G_{PD}	$4 \cdot 10^3$ V/W
G_{OPT}	$1.3 \cdot 10^{-8}$ W/Hz
G_{MIX}	0.5 V/V
G_{LPF}	

TABLE 2. Gains recap

demodulated using a mixer and a lowpass filter. Choosing the appropriate frequency and phase for the local oscillator we can select one of the two oscillating terms. If the modulation frequency is higher then the cavity linewidth ($\Omega > FSR/F$), as in our case[1], near the resonance $R_{\mathrm{cav}}(\omega \pm \Omega) \simeq -1$ thus $R_{\mathrm{phs}} = i2\,\mathrm{Im}(R_{\mathrm{cav}}(\omega)$ is purely imaginary and only the part proportional to the sine survives. The error signal will be

$$err = G_{\mathrm{PD}} G_{\mathrm{MIX}} G_{\mathrm{LPF}} 2\sqrt{P_c P_s}\,\mathrm{Im}(R_{\mathrm{phs}}) \simeq G_{PD} G_{MIX} G_{LPF} 4\sqrt{P_c P_s}\,\mathrm{Im}(R_{\mathrm{cav}}(\omega)]$$

where the last equality holds it the region where sidebands are not resonant, as can be seen in picture **??**.

As expected the error signal is linear around the region where it cross the zero. Once the lock is acquired, the frequency of the laser will be controlled in order to be always about the resonance. The linear coefficient of the error signal about the zero point, also known as optical gain, will tell us how many Watt[2] correspond to a shift of 1 Hz of the laser frequency from the resonance. Around $\omega = 0$ we have

$$\mathrm{Im}(R_{\mathrm{cav}}) = \frac{r_2(1 - r_1^2)}{FSR \cdot (1 - r_1 r_2)^2}$$

switching to $f = \omega/2\pi$ the linearised error signal will be

$$err = G_{\mathrm{PD}} G_{\mathrm{MIX}} G_{\mathrm{LPF}} \underbrace{8\pi\sqrt{P_c P_s}\frac{r_2(1 - r_1^2)}{FSR \cdot (1 - r_1 r_2)^2}}_{G_{\mathrm{OPT}}[W/Hz]} f$$

Using the parameters in tab **??** we find $G_{\mathrm{OPT}} = 1.32 \cdot 10^{-8}$ W/Hz

[1]in this case the modulation frequency is higher than the FSR, in order not to have resonant sidebands it should be $\Omega \mod FSR < dv$

[2]or Volt if we already take into account photodiode and demodulation gains

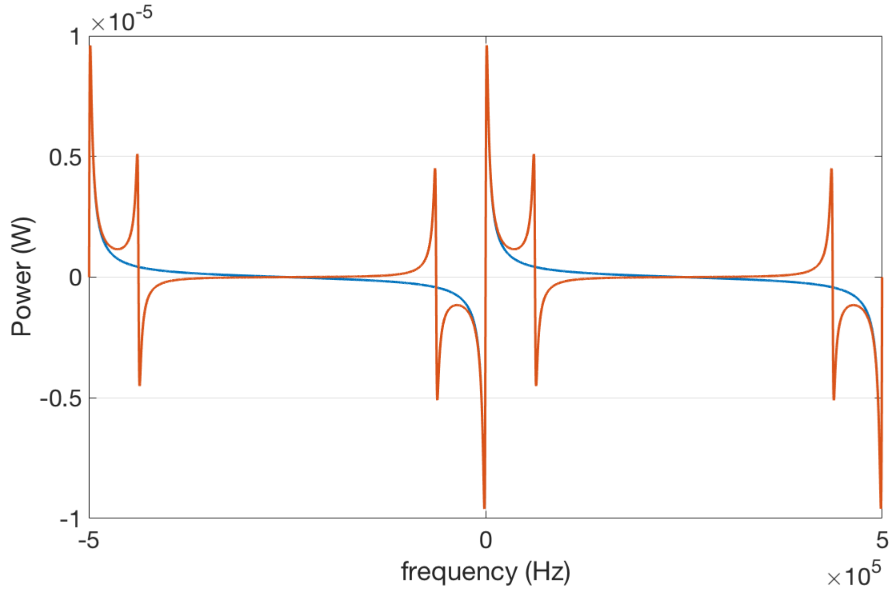

FIGURE 1. Comparison between the PDH error signal and its linearisation around zero

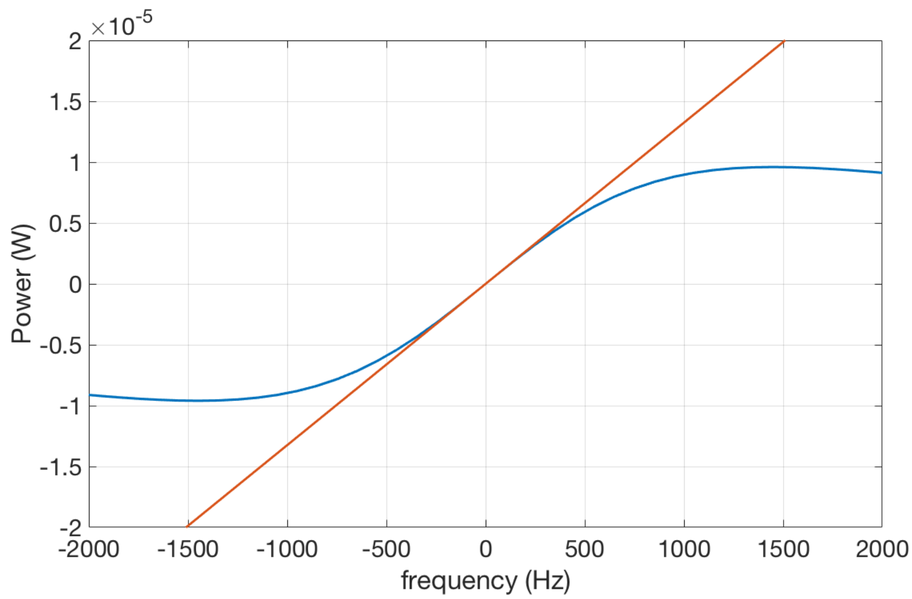

FIGURE 2. Comparison between the PDH error signal (orange line) and its approximation (blue line) assuming $R_{\text{cav}}(\omega \pm \Omega) \simeq -1$ that, as can be seen, is valid only in the regions where the sidebands are not resonant